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DIRECTED FORESTS WITH APPLICATION TO ALGORITHMS RELATED TO MARKOV CHAINS

Abstract. This paper is devoted to computational problems related to Markov chains (MC) on a finite state space. We present formulas and bounds for characteristics of MCs using directed forest expansions given by the Matrix Tree Theorem. These results are applied to analysis of direct methods for solving systems of linear equations, aggregation algorithms for nearly completely decomposable MCs and the Markov chain Monte Carlo procedures.

0. Introduction. This work is devoted to computational problems related to Markov chains (MC) on a finite state space. It is a shorten version of the author's Ph.D. thesis [Po 1]. In Section 1, using some combinatorial structures—directed forests—we present formulas and bounds for such characteristics of MCs as the stationary distribution, mean hitting times and eigenvalues of the transition matrix. These formulas and bounds have the form of rational functions of elements of the transition matrix and follow from the Matrix Tree Theorem. We apply these results to analyse four groups of algorithms.

In Section 2 we study direct methods for computing characteristics of MC which are solutions of systems of linear equations. We generalize Grassmann, Taksar and Heyman's version of the Gaussian elimination method. We give bounds for the entrywise relative error of this algorithm.

Section 3 deals with aggregation algorithms for approximation of the characteristics of perturbed MCs. Such algorithms are used to solve large

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